# Efficient Iterative Integral Technique for Computation of Fields in Electric Machines with Rotor Eccentricity

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**Abstract — To analyze the effects of small variations of the electric machine airgap due to rotor eccentricity it is necessary to compute the magnetic field highly accurately. An efficient iterative integral technique is proposed, where the material nonlinearity is treated by the polarization method, with the magnetic field determined at each iteration by superposing the contributions of the given electric currents and of the polarization. It has great advantages over the finite element based procedures, namely, the change in the rotor position does not require the construction of a new discretization mesh, very small airgaps can be taken into consideration without increasing the amount of computation, and the calculated magnetic field in the air is divergenceless and curlless, thus eliminating the introduction of spurious forces. As well, the generated phase voltages and the magnetic forces are easily calculated from the magnetic field quantities.**

## I. INTRODUCTION

Operating parameters of electric machines can strongly be influenced by the nonuniformity of the airgap, which is usually due to the rotor eccentricity. Since the airgap variations are normally very small, the analysis of the corresponding magnetic field must be performed by employing highly accurate computational methods. Recently, the finite element method has been used for this purpose [1], [2], but the rotor motion and the choice of the time steps, in the presence of eccentricity, require elaborate procedures for the construction of the discretization mesh. Very small airgaps can be accurately treated only by increasing the number of the mesh elements and, thus, the necessary amount of computation. On the other hand, in this approach one introduces spurious forces on the element boundaries, even in the air. To take into account approximately the small modifications of the airgap, an attempt has been made in [3] to apply analytic methods where, on the one hand, special boundary conditions are to be imposed in order to consider the tooth geometry but, on the other hand, the nonlinearity of the ferromagnetic material is not taken into account.

In the iterative procedure proposed in this paper, the nonlinearity of the magnetic field equations is dealt with by applying the polarization fixed point method [4],[5], the solution of the linear magnetic field problem at each iteration being obtained by using a Green function for the unbounded homogeneous space. This approach has previously been used for electric machines with uniform airgaps [6], where the difficulty related to the large dimension of the tensor matrix containing the coupling coefficients between polarizations and the average values of the magnetic induction over the elements is reduced by exploiting the pole periodicity and by using an associated periodic Green function. Such a technique cannot be applied in the case of a nonuniform airgap when the magnetic field is not periodic. In the present paper, the reduction of the number of entries into the coupling matrix, in the presence of rotor eccentricity, is achieved by using the stator tooth periodicity and the rotor pole periodicity, as well as the fact that the coupling coefficient tensors are symmetric and of zero trace.

## II. FIELD SOLUTION

The nonlinear characteristic $\boldsymbol{H} = \mathrm{F}(\boldsymbol{B})$ of the ferromagnetic material is replaced by $\boldsymbol{B} = \mu\boldsymbol{H} + \boldsymbol{I}$, where the constant $\mu$ is chosen to be everywhere equal to the permeability of free space $\mu_0$ and the nonlinearity is taken into account by the polarization $\boldsymbol{I}$,

$$\boldsymbol{I} = \boldsymbol{B} - \mu_0\,\mathrm{F}(\boldsymbol{B}) \equiv \mathbf{G}(\boldsymbol{B}) \tag{1}$$

As shown in [5], the function $\mathbf{G}$ is a contraction.

For any particular linear field problem in free space, the magnetic induction $\boldsymbol{B}$ is expressed in the form $\boldsymbol{B} = \boldsymbol{B}^J + \boldsymbol{B}^I$, where $\boldsymbol{B}^J$ is due to the given current density $\boldsymbol{J} = \boldsymbol{k}J$ and $\boldsymbol{B}^I$ is due to the polarization $\boldsymbol{I}$,

$$\boldsymbol{B}^J(\boldsymbol{r}) = \frac{\mu_0}{2\pi}\int_{D_J} J(\boldsymbol{r}')\frac{\boldsymbol{k}\times\boldsymbol{R}}{R^2}dS' \tag{2}$$

$$\boldsymbol{B}^I(\boldsymbol{r}) = \frac{1}{2\pi}\int_{D_f} \frac{(\boldsymbol{k}\times\boldsymbol{R})\{\boldsymbol{k}\cdot(\nabla'\times\boldsymbol{I}(\boldsymbol{r}'))\}}{R^2}dS' \tag{3}$$

with $\boldsymbol{k}$ being the axial unit vector. $D_J$ is the cross-sectional region where $J$ is confined, $D_f$ is the region containing soft ferromagnetic material and permanent magnets, and $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $R = |\boldsymbol{R}|$, $\boldsymbol{r}$ and $\boldsymbol{r}'$ being the cross-sectional position vectors of the observation and of the source points, respectively. $\boldsymbol{B}^J$ is calculated only once, before starting the iterative procedure. Region $D_f$ is discretised in $n_f$ elements and the polarization is taken to be constant over each element $\omega_k$, $k$=1,2,…, $n_f$. Equation (3) becomes

$$\boldsymbol{B}^I(\boldsymbol{r}) = -\frac{1}{2\pi}\sum_{k=1}^{n_f}\oint_{\partial\omega_k}\frac{\boldsymbol{k}\times\boldsymbol{R}}{R^2}(\boldsymbol{I}_k\cdot d\boldsymbol{l}'_k) \tag{4}$$

where $\partial\omega_k$ is the boundary of $\omega_k$. The average values of $\boldsymbol{B}^I$ over the mesh elements can be expressed in the form

$$\begin{pmatrix}\tilde{\boldsymbol{B}}^s\\ \tilde{\boldsymbol{B}}^r\end{pmatrix} = -\sigma^{-1}\begin{pmatrix}\overset{=ss}{\boldsymbol{a}} & \overset{=sr}{\boldsymbol{a}}\\ \overset{=rs}{\boldsymbol{a}} & \overset{=rr}{\boldsymbol{a}}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}^s\\ \boldsymbol{I}^r\end{pmatrix} \tag{5}$$

where $\sigma^{-1}$ is the inverse of the diagonal matrix of the element areas. The coupling matrix is constructed from three distinct tensor submatrices, $\overset{=ss}{\boldsymbol{a}}$, $\overset{=rr}{\boldsymbol{a}}$ and $\overset{=sr}{\boldsymbol{a}}$, corresponding, respectively, to the stator-stator, rotor-rotor and stator-rotor couplings, the first two being computed only once, whereas only the last one is corrected at each time step. The entries in these submatrices are expressed as

$$\overset{=}{\boldsymbol{a}}_{ik} = \frac{1}{2\pi} \oint_{\partial\omega_i} \oint_{\partial\omega_k} \ln R \, (d\boldsymbol{l}_i d\boldsymbol{l'}_k) \tag{6}$$

where $(d\boldsymbol{l}_i d\boldsymbol{l'}_k)$ is the dyad formed by the vectors $d\boldsymbol{l}_i$ and $d\boldsymbol{l'}_k$. The size of the coupling matrix, in the presence of rotor eccentricity, is substantially reduced by using the fact that the coupling coefficient tensors are symmetric and of zero trace, and by implementing the periodicity of the stator teeth and of the rotor poles. The magnetic field is computed by solving iteratively the nonlinear system obtained from equations (1) to (5), based on the polarization fixed point iterative technique, i.e., $\ldots \to \boldsymbol{I}_{k-1} \to \widetilde{\boldsymbol{B}}_k \to \boldsymbol{I}_k \to \ldots$

## III. MACHINE OPERATION

### A. Phase voltage

The magnetic vector potential $\boldsymbol{A} = \boldsymbol{k}A$ is determined from $A = A^J + A^I$, where $A^J$ is due to the given current density $J$,

$$A^J(\boldsymbol{r}) = \frac{\mu_0}{2\pi} \int_{D_J} J(\boldsymbol{r}') \ln\frac{1}{R} dS' \tag{7}$$

and $A^I$ is due to the polarization,

$$A^I(\boldsymbol{r}) = -\frac{1}{2\pi} \sum_{k=1}^{n_f} \boldsymbol{I}_k \cdot \oint_{\partial\omega_k} \ln R \, d\boldsymbol{l'}_k \tag{8}$$

The magnetic flux per unit axial length linking one phase is calculated as

$$\Phi = \sum_{m=1}^{M} A_m z_m \tag{9}$$

where $A_m$, $m$=1,2,…,$M$, are the vector potentials at the slot centers, $M$ is the number of machine slots, and $z_m$ is the number of conductors of the phase considered located in the slot $m$, with the direction of reference along $\boldsymbol{k}$.

### B. Magnetic forces

The resultant radial force per unit length acting on the machine rotor due to rotor eccentricity is determined by integrating Maxwell's stress tensor over an airgap surface approximated, in the 2D case, by a regular polygon with $\boldsymbol{B}$ obtained from (2) and (3).

## IV. ILLUSTRATIVE EXAMPLE

Computed and experimental results have been obtained for a small permanent magnet synchronous generator, with an airgap of 1 mm, length of 20 mm, speed of 1,500 r/min, $n_z$ =24 stator teeth and $n_p$ =2 rotor poles. The geometry of the generator and the discretization mesh used are presented in Fig.1. The field lines of the magnetic induction are also plotted in Fig.1 for an arbitrary position of the rotor. Concrete results for this electric machine will be presented in the full paper.

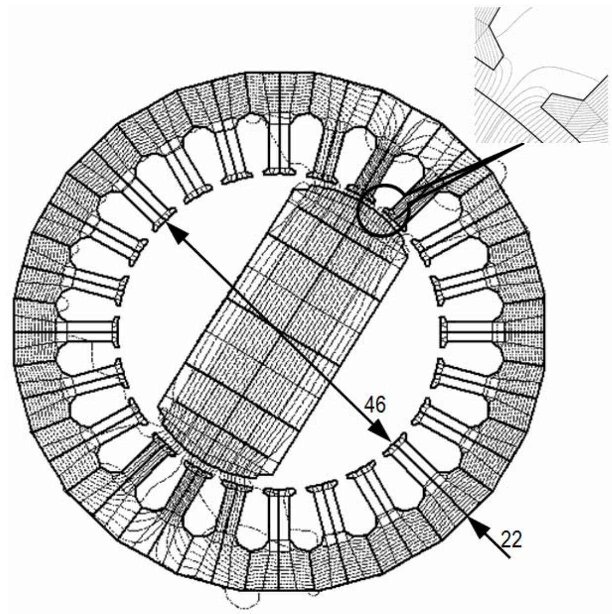

Fig. 1. Discretization mesh and field lines for a permanent magnet synchronous generator (dimensions are in mm)

## V. CONCLUSIONS

An efficient procedure is presented for a very high accuracy performance analysis of electric machines with rotor eccentricity. This procedure has substantial advantages over the finite element based procedures. Very small airgaps can be treated without increasing the amount of computation, since the discretization mesh is generated only once, using polyhedral elements, and only for the ferromagnetic bodies. As well, the calculated magnetic field in the air is divergenceless and curlless, the introduction of spurious forces being thus eliminated. The proposed method can be applied to other nonlinear magnetic devices with moving parts.

## VI. REFERENCES


[1] A. Tenhunen, "Calculation of eccentricity harmonics of the air-gap flux density in induction machines by impulse method," *IEEE Trans. on Magn.*, vol.41, no.5, pp. 1904-1907, 2005.

[2] L.S. Stephens and M.A. Casemore, "Influence of stator slot geometry and rotor eccentricity on field distribution in cylindrical magnetic actuators," *IEEE Trans. on Magn.*, vol.38, no.2, pp.1348-1356, 2002.

[3] U. Kim and D.K. Lieu, "Magnetic field calculation in permanent magnet motors with rotor eccentricity: with slotting effect considered," *IEEE Trans. on Magn.*, vol.34, no.4, pp.2253-2266, 1998.

[4] F.Hantila, "A method of solving stationary magnetic field in non-linear media", *Rev. Roum. Sci. Techn.-Electrotechn. et Energ.*, vol.20, no.3, pp. 397-407, 1975.

[5] F.Hantila, G.Preda, and M.Vasiliu "Polarization method for static fields," *IEEE Trans. Magn.*, vol.36, no.4, pp. 672-675, 2000.

[6] E.Demeter, F.Hantila, A.Kladas, and J.Tegopoulos, "A new method for magnetic field calculation in electrical machines", ICEM'94, Paris, Sept.1994.